

The Behavior of Electronic Interferometers in the Non-Linear Regime

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We investigate theoretically the behavior of the current oscillations in an electronic Mach-Zehnder interferometer (MZI) as a function of its source bias. Recently, The MZI interference visibility showed an unexplained lobe pattern behavior with a peculiar phase rigidity. Moreover, the effect did not depend on the MZI paths difference. We argue that these effects may be a new many-body manifestation of particle-wave duality of quantum mechanics. When biasing the interferometer sources beyond the linear response regime, quantum shot-noise (a particle phenomena) must affect the interference pattern of the electrons that creates it, as a result from a simple invariance argument. An approximate solution of the interacting Hamiltonian indeed shows that the interference visibility has a lobe pattern with applied bias with a period proportional to the average path length and independent of the paths difference, together with a phase rigidity.

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In the last two decades, electron interferometers became a primary tool in mesoscopic physics, for investigating quantum coherence of transport in semiconductors, and to measure and control novel quantum effects [1, 2, 3, 4, 5, 6]. Their behavior is understood so far only in the linear response regime, where a small bias is put on their sources, and the non-interacting picture is valid. In the non-linear regime several electrons are present inside the interferometer at a given time and may form, in contrast to the optical interferometers, non-trivial many-body correlations due to Coulomb interaction. Indeed, new experiments in the non-linear regime demonstrated that the Landauer-Buttiker formalism [7] seems to break down, and the interference pattern showed new peculiar behavior. Recently, an unexpected interference behavior of a Mach-Zehnder interferometer (MZI) was reported [8, 9], in which the visibility (proportional to the observed amplitude of the Aharonov-Bohm (AB) oscillations of the drains current) evolved in a lobe pattern with increasing the source bias, with zero visibility between the lobes, and a phase independence on the bias inside each lobe ("phase rigidity"). Similar visibility lobes were observed earlier in a two-terminal closed interferometers [2, 10]. In the MZI case, the lobe pattern could not be explained using any non-interacting picture, mainly because the lobes were very robust against induced asymmetry between the two path, and did not depend at all on their length difference. Recent explanations proposed for this effect using Bosonization techniques [11, 12, 13] imposed interactions between electrons that were geometry dependent, somewhat in contrast to the robustness of the experimental observation. Moreover, these propositions do not explain the total phase rigidity seen in the experiment [8].

Here we show that these lobes and phase rigidity are indeed very reasonably a non-perturbative result of the Coulomb interaction. We argue that in the non-linear regime, this interaction causes the extra electrons in-

side a two path interferometer to form a correlated state which is rooted in their quantum behavior; as electrons behaves both as particles and as waves, the phase of an interfering electron gets discrete "quantum kicks" according to the possible occupations of the additional electron states in the two arms. In some distinct source voltages those phase fluctuations cause the vanishing of the interference, which reappear at lower or higher voltages, leading to a lobe pattern in the visibility. The basis for

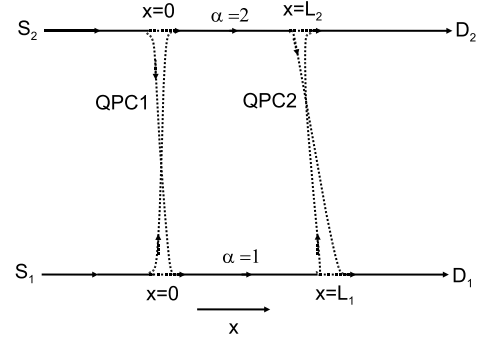


FIG. 1: Schematic of the MZI geometry, with generally asymmetric two paths.

our derivation is a general principle for mesoscopic devices, which we term Buttiker-Levinson argument (BLA): changing the chemical potential of all the sources by the same amount, should not change the transmission from any source to any drain. It is generally relevant for any energy-dependant mesoscopic devices [14], and in particular for the MZI, being two-path interferometer formed by one dimensional edge channels in the quantum Hall effect regime, since the interaction between the electrons inside the paths is unscreened by the gates nearby. Surprisingly, when applied to electron interferometers in the non-linear regime, this simple argument is violated by non-interacting models [8].

A seemingly natural solution to this inconsistency is to

correct the non-interacting models by taking into account a mean field approximation (MFA)[8, 10, 14] in which the bias induces a static mean charging in each of the interferometer arms which causes a phase shift due to Coulomb interaction. Experimentally however, this approximation failed for the MZI as the lobes were independent on the two arm length difference [8]. We now further argue that this MFA correction is never valid in the MZI, since whenever it is significant, quantum fluctuations are strong and cannot be neglected. As the phase shift must restore the BLA validity, the phase shift correction in the MZI must be given by $e^{i2\pi\langle N_1 - N_2 \rangle}$ [15], where N_α is the excess number of electrons in path α due to the source bias. Due to the applied bias this number of electrons and the phase in each arm fluctuate *binomially* (quantum shot-noise) with a large variance and, most importantly, with strong negative correlation between the two arms. Therefore the induced phase shift cannot be approximated simply by the average charging since $\langle e^{i\delta\phi(N)} \rangle \neq e^{i\langle\delta\phi(N)\rangle}$, as was suggested in [10]. Rather, in this situation of few electrons inside the MZI generally $[N, \delta\phi] \neq 0$, and non-linear interactions may cause new many-body coherent states analogous to squeezed spin states [16]. Our situation is shown below to be closely related to another non-linear system, a biased finite 1D detector channel coupled a MZI arm [17, 18].

Our starting point is the non-interacting Hamiltonian, written for spinless or spin polarized electrons in the MZI drawn in Fig. 1. It is expressed by the creation and annihilation operators $c_k^{(S_\alpha)}, c_k^{(S_\alpha)\dagger}$ for the *incoming* extended single-particle energy states from the two sources $S_{1,2}$,

$$H_0 = \sum_{\alpha=1,2} \sum_k (\epsilon(k) - \mu_{S_\alpha}) c_k^{(S_\alpha)\dagger} c_k^{(S_\alpha)}. \quad (1)$$

In this model a linear relation can be established between the *incoming* and *outgoing* k -operators from the sources and to the drains, respectively

$$c_k^{(D_\alpha)} = \sum_{\beta=1,2} s_{k,\alpha\beta}^{(MZI)} c_k^{(S_\beta)}. \quad (2)$$

where the s -matrix for the particular MZI geometry can be calculated from the product $s^{(MZI)} = s^{(QPC2)} \cdot s^{(\delta\varphi)} \cdot s^{(QPC1)}$ with the three s -matrices defined as ($a = 1, 2$)

$$s^{(QPC(a))} = \begin{pmatrix} ir_a & t_a \\ t_a & ir_a \end{pmatrix}, \quad (3)$$

$$s^{(\delta\varphi)} = \begin{pmatrix} e^{ikL_1} & 0 \\ 0 & e^{ikL_2} \end{pmatrix}$$

where r_a and t_a the reflection and transmission amplitude of QPC(a), and $L_{1,2}$ are the two MZI path lengths. We can define the annihilation operator $c_k^{(\alpha)}$ of an electron moving in arm α of the MZI as obeying the relations

$$c_k^{(\alpha)} = \sum_{\beta=1,2} s_{\alpha\beta}^{(QPC1)} c_k^{(S_\beta)} \quad (4)$$

$$c_k^{(D_\alpha)} = \sum_{\beta=1,2} \left(s^{(QPC2)} \cdot s^{(\delta\varphi)} \right)_{\alpha\beta} c_k^{(S_\beta)}.$$

Considering the local nature of the interactions and of the device, we choose to express the current in the drains using the local operators after QPC2, $\psi_{(D_\alpha)}(x, t) = \sum_k e^{ikx} c_k^{(D_\alpha)}(t)$. The resulting chemical potential at any drain D_α , μ_{D_α} , is proportional to the average current to the drain, $\mu_{D_\alpha} = \frac{\hbar}{e} \langle I_{D_\alpha} \rangle$, and since our problem is stationary in time it can be written (linearizing $\epsilon(k)$) using this operator $\psi_{(D_\alpha)}(x, t)$ at $t = 0$

$$\mu_{D_\alpha} = \frac{\hbar}{e} \langle I_{D_\alpha} \rangle = \hbar v_g \left\langle \psi_{(D_\alpha)}^\dagger(L_\alpha, 0) \psi_{(D_\alpha)}(L_\alpha, 0) \right\rangle \quad (5)$$

where v_g is the group velocity at the Fermi energy. In the non-interacting model Eq. 5 can be written as $\mu_{D_\alpha} = \mu_{0,\alpha} + \Re(\mu_{\varphi,\alpha} e^{i\varphi_0})$, with $\mu_{0,\alpha}$ a real parameter and $\mu_{\varphi,\alpha}$ a complex pre-factor of the phase ($\varphi_0 = k_F \Delta L$) dependant term. The second term is proportional to $\int_{\mu_{S_1}}^{\mu_{S_2}} \cos(\varphi_0 + \frac{\Delta L}{\hbar v_g}(\epsilon - E_F)) d\epsilon = \left[2\hbar v_g \sin(\frac{\Delta L \Delta \mu}{\hbar v_g}) \right] \cdot \cos(\varphi_0 + \frac{\Delta L}{\hbar v_g}(\bar{\mu} - E_F))$, with E_F the Fermi energy at zero bias. It violates BLA through the explicit dependance on the average source bias $\bar{\mu} - E_F$.

Since, as stated above, a MFA correction is inadequate due to the quantum fluctuations, we are forced to return to the Hamiltonian, and introduce additional non-linear Coulomb terms (whose range we assume here to be longer than the MZI arms) to take care of the BLA

$$H = H_0 + \sum_{\alpha=1,2} \frac{e^2}{2C_\alpha} N_\alpha^2 \quad (6)$$

where $N_\alpha(t) = \int_0^{L_\alpha} \rho_\alpha(x, t) dx$ is the number operator which counts the electrons which are added to the MZI path α region at time t , $\rho_\alpha(x, t) = \psi_\alpha^\dagger(x, t) \psi_\alpha(x, t)$, and $C_\alpha = \frac{e^2 L_\alpha}{\hbar v_g}$ is the electric capacitance of path α , induced by the dispersion in H_0 and Pauli principle. Hence, the added terms in the Hamiltonian do not have additional free parameters.

The role of the additional term is to locally raise the bottom of the conduction band (the electro-static potential) in path α according to its overall local charging. The effect is exactly such that when a full beam between energies E_F and $E_F + \Delta\mu$ enters the path, the last fully occupied state remains with same momentum k_F , with a new energy $\bar{\mu} = E_F + \Delta\mu$, which restores BLA validity. We should note that in addition to this dynamic effect, the new terms also add electron-hole excitations above the Fermi sea, which we ignore [19].

We shall solve now approximately the equation of motion (EOM) for the interacting case. Keeping the definition in Eq. 5 and expecting the same functional dependence of μ_{D_α} on φ_0 as in the non-interacting case, we want to show that $\mu_{\varphi,\alpha}$ develops a lobe structure behavior and phase rigidity with an energy scale independent

of ΔL . The EOM for $\psi_\alpha(x, t)$ reads

$$\begin{aligned} \frac{\partial \psi_\alpha(x, t)}{\partial t} &= \frac{1}{i\hbar} [\psi_\alpha(x), H] = \\ v_g \frac{\partial \psi_\alpha}{\partial x} - i \frac{\pi v_g}{L_\alpha} (\psi_\alpha(x, t) N_\alpha + N_\alpha \psi_\alpha(x, t)) \Theta(x) \Theta(L_\alpha - x) \end{aligned} \quad (7)$$

Where $\Theta(x)$ is the Heaviside function. We use the linear relations in Eq. 4 to define the operators $\psi_\alpha(x, t)$ at $x < 0$ and at $x > L_\alpha$

$$\begin{aligned} \psi_{(D_\alpha)}(x, t) &= \sum_{\beta=1,2} s_{\alpha\beta}^{(QPC2)} \psi_{(\beta)}(x, t), \quad x > L_\alpha \quad (8) \\ \psi_{(\alpha)}(x, t) &= \sum_{\beta=1,2} s_{\alpha\beta}^{(QPC1)} \psi_{(\beta)}(x, t), \quad x < 0 \end{aligned}$$

where $\psi_{(S_\alpha)}(x, t) = \sum_k e^{ikx} c_k^{(S_\alpha)}(t)$. Having the non-interacting solution $\psi_\alpha(x, t) = \psi_\alpha^{(0)}(x - v_g t)$ at $x < 0$ as boundary condition, Eq. 7 has a unique formal solution,

$$\psi_\alpha(x, t) = \begin{cases} \psi_\alpha^{(0)}(x - v_g t) & x < 0 \\ \mathcal{T}[U_\alpha(t)] \psi_\alpha^{(0)}(x - v_g t) \mathcal{T}^{-1}[U_\alpha(t)] & 0 < x < L_\alpha \\ \psi_\alpha(L_\alpha, t - (x - L_\alpha)/v_g) & L_\alpha < x \end{cases} \quad (9)$$

where \mathcal{T} is time-ordering operator, and we define

$$U_\alpha(t) = e^{-\frac{i\pi v_g}{L_\alpha} \int_{t-x/v_g}^t dt' N_\alpha(t')} \quad (10)$$

Eq. 9 is difficult to evaluate analytically, due to the fact that $U_\alpha(t)$ depends on $\psi_\alpha(x, t)$. However, one can obtain some general properties of the highly correlated quantum state at the drains, using the following derivation. First we approximate $U_\alpha(t)$ by replacing $\rho_\alpha(x, t)$ in $N_\alpha(t)$ with the non-interacting density $\rho_\alpha^0(x - v_g t)$. One should note that this is a very crude approximation as electrons really repel each other and change each others group velocity considerably; hence the true solution might be different in its fine details. However, note that unlike the MFA, this approximation maintains the non-Gaussian behavior of the shot-noise, as well as the negative correlation of the shot-noise between the two paths. Next, due to the fact that $N^0(t)$ are Bosonic, $[N_\alpha^0(t), N_\alpha^0(t')]$ is a C-number, so the T-ordering can be eliminated by repeatedly using Baker-Hausdorff formula, such that the phases originating from the T and the T^{-1} ordering cancel each other. Then, we take $t = 0$ and rearrange the operator in the exponent $U_\alpha = U_\alpha(t = 0) = e^{-i\phi_\alpha}$ as a weighted integral over the density operator $\phi_\alpha = \int_{-\infty}^{\infty} w_\alpha(x) \rho_\alpha^0(x) dx$. Straightforward calculation shows $w_\alpha(x)$ to be a triangle

$$w_\alpha(x) = \begin{cases} \pi(1 - |\frac{x-L_\alpha}{L_\alpha}|) & |x - L_\alpha| < L_\alpha \\ 0 & |x - L_\alpha| > L_\alpha \end{cases} \quad (11)$$

An additional approximation is to restrict the energy excitations in the operators $\rho_\alpha^0(x)$ and therefore also in the

operator ϕ_α

$$\phi_\alpha \approx \tilde{\phi}_\alpha = \sum_{k, k'=k_F}^{k_F + \Delta\mu/\hbar v_g} w_{\alpha, k-k'} c_{\alpha, k}^\dagger c_{\alpha, k'} \quad (12)$$

where $w_{\alpha, k-k'}$ is the Fourier transform of $w_\alpha(x)$, and \tilde{O} denote an operator restricted to the energies $[E_F, E_F + \Delta\mu]$. This approximation is very intuitive as it takes into account only dephasing due to excess shot-noise, disregarding virtual transitions to energies above $E_F + \Delta\mu$. It was proven very useful in explaining the unique experimental results of dephasing by shot-noise [17]. With these approximations, the pre-factor of the phase dependant part of Eq. 5 reads

$$\mu_\varphi = \hbar v_g \langle \Psi_0 | \tilde{U}_1^\dagger \tilde{U}_2 \tilde{\psi}_1^{0\dagger}(L_1) \tilde{\psi}_2^0(L_2) \tilde{U}_1 \tilde{U}_2 | \Psi_0 \rangle \quad (13)$$

where $|\Psi_0\rangle = \left(\prod_{k=k_F}^{k_F + \Delta\mu/\hbar v_g} c_k^{(S_1)\dagger} \right) |gs(\mu_{S_{1,2}} = E_F)\rangle$. Eq. 13 can be evaluated by noting that $\tilde{\phi}_1$ and $\tilde{\phi}_2$ commute, so one can introduce a basis of a complete set of eigenstates of the two operators $\{|\varphi_1, \varphi_2\rangle\}$. Note that the single-particle matrix $w_{\alpha, k, k'}$, restricted to the voltage window $[E_F, E_F + \Delta\mu]$, has a discrete set of non-vanishing eigenvalues, corresponding to localized electron-states near the finite influence region of $w(x)$ (Eq. 11). The eigenstate $|\varphi_\alpha\rangle$ of the operator $\tilde{\phi}_\alpha$ is a specific choice of occupations of those single-particle states, the value φ_α being the sum of the eigenvalues of the occupied states. With two insertions of such a full set, Eq. 13 now reads

$$\begin{aligned} \mu_\varphi &= \hbar v_g \sum_{\varphi_1, \varphi_2, \varphi'_1, \varphi'_2} \langle \Psi_0 | \varphi_1, \varphi_2 \rangle \langle \varphi_1 | \tilde{\psi}_1^{0\dagger}(L_1, 0) | \varphi'_1 \rangle \langle \varphi'_1 | \\ &\cdot \langle \varphi_2 | \tilde{\psi}_2^0(L_2, 0) | \varphi'_2 \rangle \langle \varphi'_2 | \varphi'_1, \varphi'_2 | \Psi_0 \rangle \cdot e^{i(\varphi_1 - \varphi_2 + \varphi'_1 - \varphi'_2)} \end{aligned} \quad (14)$$

This expression differs from the non-interacting one only by the exponent at the end of the r.h.s. of the equation. Eq. 14 can be evaluated numerically, through diagonalization of the matrices $w_{\alpha, k, k'}$. Fig. 2 shows the result for the visibility, as a function of $\langle N \rangle = \langle N_1 + N_2 \rangle = \frac{\bar{L}}{\hbar v_g} \Delta\mu$, at $\frac{\langle \Delta N \rangle}{\langle N \rangle} = \frac{\Delta L}{L} = 0.2$, and $|t_1|^2 = |t_2|^2 = 0.5$. The prediction of the non-interacting model is also plotted for comparison. One can clearly see that while in the non-interacting model the visibility and phase are slowly varying (because the scale on which they change is proportional to ΔL^{-1}), in the interacting model a lobe pattern appears in the visibility, with a stick-slip behavior of the phase. The lobes evolution is much faster, apparently proportional to $\langle N \rangle^{-1}$ and hence to \bar{L}^{-1} , in agreement with the experimental results. Fig. 3 shows the evolution of the eigenvalues of the single-particle matrices w_1 and w_2 as a function of $\langle N \rangle$. At small bias there are two non-zero eigenvalues, one for each of the phase operators. When put in the exponents of Eq. 14, they exactly cancel the kinetic phase induced from H_0 , resulting in

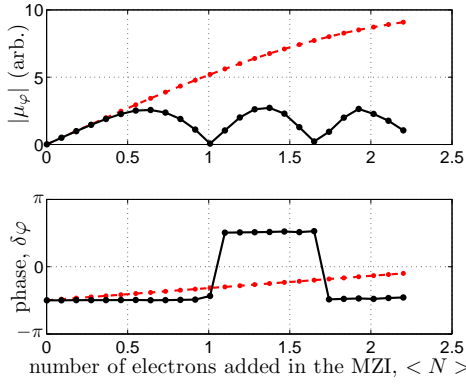


FIG. 2: The amplitude and phase shift of the interference oscillations in the chemical potential of the MZI drain, as a function of $\langle N \rangle = \bar{L}\Delta\mu/hv_g$ in the non-interacting model (red curves) and in presence of the interaction (Eq. (14), black curves), for $\langle \Delta N \rangle / \langle N \rangle = \Delta L / \bar{L} = 0.2$. The lines between the calculated points are a guide to the eye. Note that we plotted here the total current visibility, which is more suitable for analyzing the lobes than the differential visibility [9].

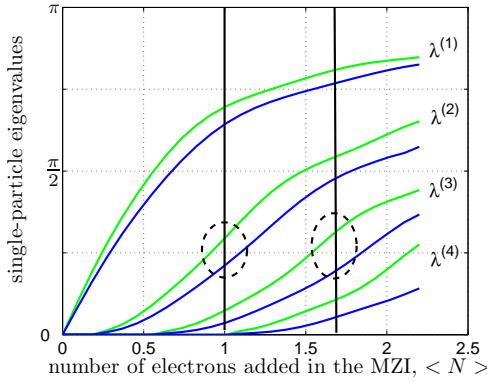


FIG. 3: The pairs of the single-particle eigenvalues of the phase operators w_1 and w_2 of the two paths (blue and green lines), as a function of $\langle N \rangle$, for $\langle \Delta N \rangle / \langle N \rangle = 0.2$. Black lines mark the zero visibility in Fig. (2). Note that at every such zero, there is a pair of eigenvalues whose sum is $\pi/2$.

phase rigidity. The first zero of the visibility occurs at $\langle N \rangle = 1$. As seen from Fig. 3, at this value the sum of the second pair of single-particle eigenvalues of the two arms is exactly $\lambda_1^{(2)} + \lambda_2^{(2)} = \pi/2$. This can be understood, by assuming that the operators $\tilde{\psi}_1^{0\dagger}(L_1, 0)$ and $\tilde{\psi}_2^0(L_2, 0)$ in Eq. 14 act effectively only on the first single-particle state (the numeric calculation indeed shows that the contribution of the other electron transitions in Eq. 14 is negligible). However the phase of these two operators is modulated by $\varphi^{(1)} - \varphi^{(2)} + \varphi'^{(1)} - \varphi'^{(2)}$, which contains also the occupation of the second eigenstates. As the second electron can occupy the lower arm or the upper arm, a zero visibility happens when the difference in the added phase between these two options become

π , so they coherently cancel each other. Quite generally, the visibility goes to zero whenever a sum of a pair of a single-particle eigenvalues of the two paths reaches $\pi/2$.

Compared with the experiment, the theory still has shortcomings. The first is that the visibility lobes do not have the overall decaying envelope. Moreover, differentiating the curve in Fig. (2) near $\langle N \rangle = 1$, leads to a phase-dependant differential response $d\mu_{D,\varphi}/d\mu_{S_1}$ which is twice that of the linear response; a differential visibility of 200%. These might be a nonphysical result of our approximations, which will be absent in the exact solution of Eq. 7.

In conclusion, in this paper we investigated the effect of quantum shot-noise inside electronic interferometers on the interference visibility in the non-linear regime. It was well established that an interaction Hamiltonian such as in Eq. 6 must always replace the non-interacting one, which quite generally leads to lobe pattern of the visibility with increasing source bias. It would be desirable to apply the theory developed here to other mesoscopic devices such as the two-terminal AB interferometers [2, 10], the MZI working in the fractional quantum Hall regime, or the two-particle interferometer [5, 6].

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